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MATHEMATICS.—*The reduction of all physical dimensions to those of space and time.* A. P. MATHEWS,[1] University of Cincinnati.

To reduce the number of different kinds of things has been the general course of development of science. Thus in chemistry the diverse substances on the earth's surface have been found to be composed of about 100 simpler substances or elements; and these elements, in their turn, of different numbers of positive and negative electrons. Similarly the forms of radiant energy—heat, light, X-rays, electromagnetic waves—have all been reduced to one kind differing only in size. So also in the case of those other entities with which physics deals—electricity, magnetism, force, energy, matter, heat—increase of knowledge has enabled a considerable simplification accompanied by a clarification of ideas.

The end of this movement must be to express the physical entities of the universe in the terms of the four dimensions, three of space and that of time. With these four dimensions we should then be able to write the whole of the physical universe in the equations of a four dimensional space.

There is, to be sure, a fifth dimension, which we must at present conserve, namely, the unknown dimensions of psychism. By psychism I mean that property of matter, at present neglected by the physicist, which is exhibited in its clearest form in living things and which shows itself in thought and consciousness in such large psychic units as ourselves. The course of evolution of living beings has been such as to create larger and more perfect psychic units. We might formulate the general law that the course of evolution was in the direction of increase of psychism. If we put P for the undetermined dimensions of psychism,—the unknown, and usually unrecognized, psychic factor

[1] Received, April 12, 1923.

in all matter and energy—the five dimensions in which the universe as at present known to us could then be described would be $(L_1) \times (L_2) \times (L_3) \times (T) \times (P)$. It may be possible in the future to write (P) in terms of space and time. Indeed it may possibly be nothing else than time, since, as has been generally recognized, time has a definite psychical element in it.

Leaving the dimensions of P at one side, the object of the present paper is to show how all the physical phenomena of the world may be expressed in dimensions of space and time to the great simplification and clarification of many conceptions.

Several attempts to do this have been made already with more or less success. Sir Oliver Lodge,[2] for example, has pointed out that the

[2] Lodge: Modern Views of Electricity, 1889, Macmillan & Co. Appendix, page 402.

"Comparing many electrical equations with corresponding mechanical ones we find that the product LC (L being length and C current) takes the place of momentum(mv) and that $1/2\ LC^2$ takes the place of kinetic energy ($1/2\ mv^2$) and indeed is the energy of a current. Hence it is natural to think of L as involving inertia and of μ and $4\pi\mu$ as a kind of density of the medium concerned. Assuming this $4\pi/K$ at once becomes an elasticity coefficient (as indeed electrostatics suggests) because $\mu K v^2 \equiv 1$; and the dimensions of all electrical units can be specified as follows, without any arbitrary convention or distinction between electrostatic and electromagnetic units.

$$\text{Sp. ind. cap., } K = \frac{\text{strain}}{\text{stress}} = \frac{\text{area}}{\text{force}} = \frac{LT^2}{M} = \text{shearability}$$

$$\mu = \frac{\text{inertia}}{\text{volume}} = \frac{M}{L^3} = \text{density}$$

$$\text{Electric charge} = L^2 = \frac{\text{volume}}{\text{displacement}}$$

$$\text{Magnetic pole} = \frac{M}{T} = \text{momentum per unit length}$$

$$\text{Electric current} = \frac{L^2}{T} = \text{displacement} \times \text{velocity}$$

$$\text{Magnetic moment} = \frac{ML}{T} = \text{momentum}$$

$$\text{E. M. F.} = \frac{\text{work}}{Q} = \frac{M}{T^2} = \text{pressure} \times \text{displacement, or work per unit area}$$

$$\text{Intensity of magnetic field, } H = \frac{F}{m} = \frac{L}{T} = \text{velocity}$$

$$\text{Intensity of electrostatic field, } \frac{F}{Q} = \frac{M}{LT^2} = \text{energy per unit volume}$$

$$\text{Surface density} = \text{a pure number}$$

$$\text{Electric tension} = \frac{M}{LT^2} = \text{a pressure or tension}$$

dimensions of (e), quantity of electricity, may be (L^2). Lewis and Adams[3] in the manner of the relativists reduced space and time to one dimension, an interval, I; but kept mass, M, as a dimension. They thus equate time with length. On this basis and disregarding specific inductive capacity which now, with velocity, has no dimensions, they attempt to explain several exact numerical agreements they have found between various constants. These agreements will be considered in a later paper.

In all these attempts to reduce the number of dimensions, mass has been the principal stumbling block. The discovery of the electrical constitution of matter enables us, however, to write mass in terms of electricity and self induction and reduce it to the dimensions of space, or (L^3). Fournier d'Albe[4] has already shown how the dimensions of magnetism may be written in those of electricity, but he still keeps electric quantity, or (e), as a fundamental unit along with mass. His identification of magnetism with electricity in motion, however, practically involves the conclusion that magnetic permeability, μ, which is generally considered to be a density, has no dimensions, and that accordingly mass and space are identical.

Since electricity appears to be the most fundamental quantity of all, the first step is to reduce it to the dimensions of L^2.

I. THE DIMENSIONS OF QUANTITY OF ELECTRICITY, (e)

Matter is composed of electrical charges, the atoms being essentially electrical doublets; positively charged nuclei or spheres, accompanied by an equal quantity of negative electricity, negative electrons.

By matter we express three fundamental concepts, namely space, weight and inertia. Matter is that which occupies space, and possesses

Capacity, $S = \frac{Q}{E} = \frac{L^2T^2}{M}$ = displacement per unit pressure

Coefficient of resistance $= \frac{E}{C} = \frac{M}{L^2T}$ = impulse or momentum per unit volume

Magnetomotive force $= 4\pi nC = \frac{L^2}{T}$ = current

Reluctance $= 1/\mu A = L^2/M$ = area/inertia

Magnetic induction, $I = M/T$ = moment of momentum per unit area

Coefficient of induction (self or mutual) $1/C = M/L^2$ = inertia per unit area

[3] Lewis and Adams: A Theory of Ultimate Rational Units; numerical relations between elementary charge, Wirkungs quantum, constant of Stefans Law. Physical Review, 1914, Ser. 2, III, p. 92.

inertia and weight. Matter or mass has three spatial dimensions and it is from it that we get our conception of 3 dimensional space. The recognition that matter consists of electric charges carries with it the consequence that inertia is self induction. For to move electrical charges, or to change their rate of motion, work, positive or negative, must be done. Inertia is hence nothing but self induction. Every time an object is moved a myriad of parallel electric currents are produced. The self induction, or inertia, is due presumably to the production of a stress, strain or twist in the ether. This ether stress, flux or twist per second is what is known as energy.

Mass is then proportional to quantity of electricity and self induction.

(1) Mass = Quantity of electricity × self-induction.

(2) $M = (e) \times I$

Substituting the dimensions for these quantities:

(3) $$(M) = (M^{1/2}L^{3/2}T^{-1}K^{1/2} \times L\,\mu)$$

and since $$\mu^{1/2}K^{1/2} = \frac{T}{L}$$

(4) $$(M) = \left(M^{1/2}L^{3/2}T^{-1} \times \frac{T}{L} \times L\mu^{1/2}\right) = (M^{1/2}L^{3/2}\mu^{1/2})$$

(5) $$\therefore (M) = (L^3\mu)$$

(6) $$(\mu) = \left(\frac{M}{L^3}\right)$$

In other words μ, magnetic permeability, has the dimensions of a density as recognized by Maxwell, Lodge and Williams.

(7) $$\text{Since } (\mu K) = \frac{T^2}{L^2}$$

(8) $$\therefore (K) = \left(\frac{LT^2}{M}\right) = \text{specific inductive capacity}$$

Substituting this value of K in the dimensions of (e)

(9) $$(e) = (M^{1/2}L^{3/2}T^{-1}K^{1/2}) = \left(M^{1/2}L^{3/2}T^{-1}\frac{L^{1/2}T}{M^{1/2}}\right) = (L^2)$$

Quantity of electricity, or (e), has therefore the dimensions of L^2, as already pointed out as a possibility by Lodge.[2]

II. THE DIMENSIONS OF MAGNETISM OR MAGNETIC FLUX

The electrical theory of magnetism shows that this is nothing else than the phenomenon of electricity in motion. The quantity of it, that is (M′), or magnetic flux, is determined by, or is nothing else than, the quantity of electricity multiplied by its velocity. Hence the dimensions of magnetic flux (M′)

$$(M') = (e) \times (v) = \left(L^2 \times \frac{L}{T}\right) \tag{10}$$

$$\therefore (M') = \frac{L^3}{T} \tag{11}$$

Magnetism is space per second; or it is electric quantity × velocity; or it is electric current, $\left(\frac{L^3}{T}\right)$, times a length; or it is current, $\left(\frac{L^3}{T}\right)\times$ inductance (L); or mass × a frequency.

III. THE DIMENSIONS OF MASS

The dimensions of mass may be obtained from those of magnetism.

$$(M') = (M^{1/2}L^{3/2}T^{-1}\mu^{1/2}) \tag{12}$$

Substituting for μ its dimension $\frac{M}{L^3}$ in (6)

$$(M') = \left(M^{1/2}L^{3/2}T^{-1}\frac{M^{1/2}}{L^{3/2}}\right) = \left(\frac{M}{T}\right) \tag{13}$$

That is magnetism is mass per second. Or in another way, mass (M), is nothing else than magnetic flux-seconds. But as shown in (11) magnetic flux is the product of electric quantity by velocity, or $(M') = \left(\frac{L^3}{T}\right)$. Hence:

$$\left(\frac{L^3}{T}\right) = \left(\frac{M}{T}\right) \tag{14}$$

$$\therefore (M) = (L^3) \tag{15}$$

Mass, therefore, has three dimensions of space. This follows also from the dimensions of μ, magnetic permeability. This, as has been shown, is a density, or $\left(\frac{M}{L^3}\right)$, and it is usually identified with the density of the ether (Maxwell, Lodge, Tunzelman, etc.). Since the ether is

incompressible, its density is everywhere the same. Consequently mass may be identified with volume of ether. It has the dimensions of space. This is in accordance with the conclusions of Fournier d'Albe[4] who gives μ no dimensions and also with the identification of matter with space by Descartes. Since mass is magnetic flux-seconds, its relation to the quantum constant, h, which is ergs-seconds, is apparent at once, and this will be taken up later. Since mass is magnetic flux-seconds and magnetic flux is quantity of electricity $\times$ velocity, mass is current $\times$ inductance $\times$ time, or it is, as specified at the outset, proportional to quantity of electricity $\times$ inductance.

The derivation of the dimensions of mass can also be made from energy in the following way:

Energy consists of the product of two factors: a quantity factor, and an intensity factor. The total work it is capable of doing depends on these two factors. Whether any interchange of energy occurs between two systems depends always on the intensity factor, not on the quantity factor. The quantity factor of energy always has the dimensions of mass, (M); the intensity factor has the dimensions of L^2/T^2, that is the dimensions of the square of a velocity, or an elasticity. For example, the total energy in a waterfall is measured by the quantity of water available multiplied by the height of the fall and the acceleration due to gravity. The intensity factor is hence height, or (L), times an acceleration or (L/T^2) and this is equal to (L^2/T^2). These dimensions must be the same for the intensity factor of every form of energy; and from them the dimensions of mass may be obtained. For example, volume energy as it may be called, i.e., the total kinetic energy of the molecules of a gas, is proportional to the product of the pressure per square centimeter multiplied into the volume. The intensity factor here is the pressure per square centimeter, which decides whether energy will flow from one to another of two containers put into communication. The intensity factor is hence $F/L^2 = M/LT^2$. But this is equal to L^2/T^2. Hence $(M) = (L^3)$. Similarly the quantity factor in the case cited is the volume, (L^3), and this is necessarily equal to (M). Similarly the intensity factor of free energy, the energy of radiation for instance, is the density of the energy, or the energy per cc., $ML^2/L^3T^2 = M/LT^2 = L^2/T^2$. Hence again $M = L^3$. Temperature is often called the intensity factor of heat energy, but this is incorrect if temperature be identified with the kinetic energy of the molecules. The temperature of a molecule is its kinetic energy. The

[4] Fournier: The Electron Theory. Longmans, Green & Co. 1906.

intensity factor, however, is again the pressure this molecule can exert per square centimeter, or M/LT^2, and this being equal to L^3/T^2, $M = L^3$. The intensity of an electrostatic field is M/LT^2; from whence again $M = L^3$.

Hence since everywhere intensity of energy has the same dimensions, L^3/T^2, the dimensions of mass are found to be L^3, just as they were found from the electrical constitution of matter.

The great gain made in reducing mass and electric quantity to the dimensions of (L^3) and (L^2) respectively will be apparent. If we keep mass and electricity fundamental dimensions, (M) and (E), as Fournier does, there is no indication of any relation between them or of their real meaning. But if we write mass as (L^3), it is clear at once that it represents space or volume, that it consists of three components, and in addition that it contains electric quantity, or (L^2) and a length or self induction, (L). The relation between mass and electricity is seen at once. Similarly, writing electricity as (e), exhibits nothing of its qualities, whereas (L^2) makes it a surface phenomenon, and relates it at once to its peculiarity of accumulating on surfaces and to its surface density. The relations between quantity of magnetism $\left(\frac{L^3}{T}\right)$ electricity (L^2), and mass (L^3), are clear by simple inspection. For example, magnetic flux is (L^2/T), while electric current is (L^2/T). This shows at once that magnetic flux is the product of current by its length. And electric quantity or, L^2, is magnetic flux $\times$ (T/L). That is, it is magnetic flux times the square root of K, the specific inductive capacity. The specific inductive capacity has the dimensions of the ratio of electric quantity to quantity of magnetic flux. The results of considering T equivalent to L_4 are considered on page 206.

IV. THE DIMENSIONS OF THE ETHERIAL CONSTANTS

Since μ, magnetic permeability, $= \frac{M}{L^3} = \frac{L^3}{L_3} = 1$ and so has no dimensions, and $(\mu\ K) = \left(\frac{T^2}{L^2}\right)$, K being the specific inductive capacity:

$$\therefore (K) = \left(\frac{T^2}{L^2}\right) \tag{16}$$

These are the dimensions already ascribed to K by Fournier d'Albe. And as he has pointed out, all the phenomena of refraction and disper-

sion are readily comprehended if K has these dimensions. In other words, K is inversely as current × a frequency; or inversely as electric quantity × angular acceleration; or it is inversely as the square of a velocity, the velocity being the velocity of long etherial waves in the medium.

All of these have a meaning. For example K being inversely as electric quantity × angular acceleration would mean that K was directly proportional to the square of the number of electrons per cc. $= \frac{1}{L^2}$ and 1 ÷ angular acceleration might be the "laxity" of the electrons. Also as $\frac{L^2}{T^2}$ is elasticity, K is inversely as an elasticity.

While the numerical value of μ is very nearly unity, except in paramagnetic and ferromagnetic substances, K varies from unity to 80 in dielectrics and to infinity in perfect conductors in which electromagnetic waves will be stopped.

The other great etherial constant is that of *gravitational permeability*, G. The numerical value of this constant for all known substances is so near unity that the constant itself is usually omitted from the formulas of gravitational attraction. And accordingly it is usually assumed that the gravitational attraction of two masses does not depend on the medium. This, however, must be an incorrect assumption if electromagnetic mass is also gravitational, as is probable. Electromagnetic or electrostatic attractions involve the medium, and as matter is nothing but electrical charges its mutual attraction must so depend also.

The Newtonian law of attraction shows this necessity also.

The following reasoning equally indicates this.

While electric quantity, or (e), is independent of the medium, the force between charges, i.e., the attraction or repulsion of two charges, depends upon the medium. It depends on the specific inductive capacity, or K. Similarly while quantity of magnetism, or magnetic flux, that is electricity in motion, does not depend on the medium, the force between two poles or two currents does so depend. It involves μ, the magnetic permeability. Just so in the case of mass. While the quantity of mass is independent of the medium, the force between two masses, which is called gravitation, or weight, must depend on the medium. This is a necessary consequence of the electrical constitution of matter. Gravitation must involve a quality of the medium which we may call the gravitational permeability and designate by G.

This may be expected to involve both the specific inductive capacity and magnetic permeability. To get its dimensions we have to turn to the Newtonian law of gravitation.

$$F = \frac{M\,M'\,g}{d^2} \tag{17}$$

Equation (17), which states that the force of attraction is directly as the product of the masses and inversely as the square of the distance, leaves out the gravitational permeability and accordingly does not balance dimensionally. The equation is ordinarily made to balance dimensionally by arbitrarily ascribing to g, the dimensions $\frac{L^3}{MT^2}$.[5] But this is incorrect, for g is a numerical constant without dimensions, having the value of 6.66×10^{-8} and is in the equation because the unit of mass has been arbitrarily chosen as that of 1 cc. of water at a certain temperature. Had unit mass been defined as that mass exerting unit force on another similar mass at unit distance, g, would have had the value of unity and would not have been in. It is clear that the factor G, gravitational permeability, must be put into the denominator. Hence the equation would be:

$$F = \frac{M\,M'\,g}{G\,d^2} \tag{18}$$

and G will have the dimensions

$$(G) = \left(\frac{M\,T^2}{L^3}\right) = (\mu T^2) = eK\mu = L^2 \times \frac{T^2}{L^2} = T^2 \tag{19}$$

That is, gravitational permeability is equal to magnetic permeability $\times$ specific inductive capacity $\times$ electric quantity or area. Its final dimension is (T^2), which is either the square of a period, or the reciprocal of angular acceleration. The value of G will probably be unity for all substances except possibly hydrogen, for hydrogen alone is exceptional in its atomic weight. All other elements have atomic weights which are whole numbers, the weight of oxygen being taken as 16. Hydrogen alone has an atomic weight which is not a whole number. Its atom instead of a weight of 1, is 1.008. It is not probable that this is due to the presence of isotopes. It may be attributed to the fact that the gravitational permeability of hydrogen is less than unity and probably in the proportion $\left(\frac{1}{1.008}\right)^2$. Harkins has ascribed the difference

[5] See for example Planck. Heat Radiation p. 174–175. (Translation by Masius.)

between the weight of hydrogen when free and hydrogen as a constituent of other elements to a "packing effect," but it is possibly more natural to ascribe it to a variation in gravitational permeability.

V. THE NATURE OF ENERGY

The foregoing reduction of physical quantities to space and time makes clear the nature of energy, which has been so difficult a conception to grasp.

$$(20) \qquad (E) = \text{energy} = \left(M \times \frac{L^2}{T^2}\right);$$

but since $M = L^3$

$$(21) \quad (E) = \left(\frac{L^5}{T^2}\right) = \left(\frac{L^3}{T} \times L^2 \times \frac{1}{T}\right) = \text{magnetic flux} \times \text{electric quantity per second.}$$

In other words all energy, of whatever kind, potential or kinetic, is the product of magnetic flux and electric current, or magnetic flux times electric quantity per second or of magnetic flux times electric quantity $\times$ a frequency. This magnetic flux may be identified, as is usual, with ether flow (that is electric quantity $\times$ velocity), and electric quantity with ether twist. Hence all energy is in the ether and is nothing else than a certain quantity of twisted etherial flow or strain.

That potential energy is nothing else than some kind of strain in the ether, the latter being perfectly elastic, is generally supposed. So all the energy of position whether this be gravitational energy, as in the separation of masses; electrostatic, as in the separation of charges; chemical, as in the separation of atoms; cohesional, as in the separation of molecules; or magnetic, as in the separation of magnetic poles or currents, is nothing else than a strain, twist or what not in the ether. It is an etherial phenomenon. It is always and everywhere magnetic flux $\times$ electric quantity $\times$ frequency. Similarly with kinetic energy; for kinetic energy is the expression of inertia and elasticity. Inertia is self induction. To move an electric charge from rest, or to increase its velocity when in motion, strain in the ether is set up or increased just as in separating unlike charges to make potential energy. The greater the velocity, and the greater the mass, i.e., the greater the number of electric charges moved, the greater is this strain. When a moving body or current stops, or is retarded, this strain or energy or deformation is suddenly released. Consequently all energy whether

kinetic or potential is etherial strain. And quantity of energy is quantity of etherial distortion per second.

Now both magnetic flux and electric quantity are often particulate. Magnetic flux × length is so many magnetons; electric quantity, so many electrons. An electron is the smallest possible amount of electricity which can exist independently; and a magneton is possibly the smallest amount of magnetic moment. Hence energy which is nothing more than the quantity of magnetic flux and electric quantity formed in the ether per second by moving particulate electric charges may also be supposed to be particulate. It must be in multiples of "kinetons." And we may identify one "kineton" with one quantum or several quanta of energy.

The quantum constant h has the dimensions of ergs-seconds. It is, therefore, nothing else than a definite amount of magnetic flux × electric quantity.

$$(22) \qquad (h) = \text{ergs-seconds} = \frac{L^5}{T^2} \times T = \left(\frac{L^5}{T}\right) = \left(\frac{L^3}{T} \times L^2\right)$$

Hence h is related to mass. Mass is magnetic flux seconds and h is ergs-seconds—that is, it is magnetic flux × electric current seconds.

Temperature, θ, has the dimensions of energy. It too is (L^5/T^2), or magnetic flux × electric charge per second. Temperature is nothing else than the product of magnetism by current. This is obvious whether we are dealing with temperature of a material body which is the kinetic energy of its molecules and atoms, or the temperature of the ether, radiant heat, which is nothing else than an electro-magnetic disturbance in the ether; i.e., a certain quantity of free magnetism × electric current; or latent heat which is potential energy.

This conception that all energy, of all kinds, whether potential or kinetic, is nothing else than a certain quantity of magnetic flux multiplied by a certain quantity of electric current in the luminiferous ether, the writer has found to be extremely useful in simplifying his conceptions. It is of course an old conception and one which should be revivified. But if magnetic flux is nothing but a phenomenon of electricity in motion, energy may be represented as $\frac{L^5}{T^2} = \frac{L^2}{T} \times \frac{L^3}{T} \times L$. That is, energy is the product of the square of electric current in the ether, multiplied by self induction, or by a length; or it is the square of electric quantity multiplied by acceleration $L^2 \times L^3 \times \frac{L}{T^2}$.

Of these conceptions the most useful appears to the writer to identify magnetic flux with etherial flow, and electric charge with a twist, as is usual.

Having identified the quantum constant, h, with the product of a certain quantity of magnetic flux by electric quantity which when multipled by a frequency gives energy, one is tempted to go farther and identify h, *with the total minimum quantity of magnetic flux* $\times$ *electric quantity in a wave front.* This quantity is the same in every wave, whatever its period, which is set up by one electron. If this quantity is multiplied by the number of such waves per second, we have energy, that is the amount of magnetic flux times electric quantity per second radiated by that electron.

Energy is radiated in quanta since no energy is radiated until an amount of magnetic flux and electric quantity equal to that contained in a wave front due to one electron is reached. This quantity of strain must be accumulated before radiation can occur. A wave or pulse is then set travelling through the ether. It is as if a slip occurred when this quantity accumulated. Since the wave front contains mass, (L^3), or magnetic flux multiplied by time, the waves will exert pressure, and will be attracted gravitationally.

The relations of the numerical value of h, to other constants of nature will be considered in a following paper.

Since energy radiated is hn, depending only on frequency, it would seem to follow that the amplitude of every wave from a single electron must be the same at its origin. An electron accelerated by the absorption of energy cannot change its amplitude of radiation. All that happens by the acceleration is an increase in frequency, a shifting in other words toward the shorter waves. It is an interesting question what this fundamental amplitude is. An increase in amplitude could then only be brought about by the fusion of waves; by an increase in the number of oscillators, in other words.

For convenience table 1 gives the dimensions of all entities in those of space and time.

VI. THE DIMENSIONS OF ALL NATURAL PHYSICAL PHENOMENA IF TIME IS A FOURTH SPATIAL COORDINATE, L_4

In the simple way just stated we have reduced the dimensions of all physical phenomena to those of space, (L_1, L_2, L_3) and time, (T). If now, time be considered a 4th spatial coordinate and equated with L, being written L_4, and if we consider this four dimensional space to be isotropic, so that all four directions are equivalent, as is done in

TABLE 1

Time	T	Elasticity $ML^{-1}T^{-2}$	$\frac{L^2}{T^2}$
Period	T		
Frequency	$\frac{1}{T}$	Surface density $\frac{E}{L^2} = \frac{L^2}{L^2} =$	1
Angular velocity	$\frac{1}{T}$	Current	$\frac{L^2}{T}$
Angular acceleration	$\frac{1}{T^2}$	Electric repulsion (force between charges)	$\frac{L^4}{T^2}$
Gravitational permeability	$G = T^2$	Gravitation attraction, $M^2L^{-2}G^{-1} =$ (force between masses)	$\frac{L^4}{T^2}$
Length	L		
Area	L^2	Magnetic attraction $M'^2L^{-2} =$ (force between poles) (force between circuits)	$\frac{L^4}{T^2}$
Electric quantity (e)	L^2		
Velocity	$\frac{L}{T}$	Electric field	$\frac{L^2}{T^2}$
Acceleration	$\frac{L}{T^2}$	Potential	
Mass, m,	L^3	Coefficient of self induction	L
Magnetic flux, M',	$\frac{L^3}{T}$	Magnetic Pole	$\frac{L^2}{T}$
Force, MLT^{-2}	$\frac{L^4}{T^2}$	Magnetic Moment	$\frac{L^4}{T}$
Moment MLT^{-2}	$\frac{L^4}{T^2}$	E. M. F. $=$	$\frac{L^2}{T^2}$
Work ML^2T^{-2}	$\frac{L^5}{T^2}$	Intensity of magnetic field	$\frac{L}{T}$
Energy ML^2T^{-2}	$\frac{L^5}{L^2}$	Intensity of electrostatic field	$\frac{L^2}{T^2}$
Heat, Θ	$\frac{L^5}{T^2}$	Surface density = a pure number	
Power	$\frac{L^5}{T^3}$	Capacity	$\frac{T^2}{L}$
Density $\frac{M}{L^3}$	1	Coefficient of resistance	$\frac{1}{T}$
Pressure per sq. cm. $ML^{-1}T^{-2}$	$\frac{L^2}{T^2}$	Magnetomotive force	$\frac{L^2}{T}$
K, sp. ind. cap.	$\frac{T^2}{L^2}$	Reluctance	$\frac{1}{L}$
μ = magnetic permeability	1	Magnetic induction	$\frac{L^2}{T}$
Moment of inertia, ML^2	L^5		

relativity, then various interesting results follow, and these may now be briefly considered. This enables us to write all dimensions as L or its powers. In table 2 the dimensions of all entities are given in terms of L.

TABLE 2

Moment of inertia	L^5	Angular velocity	$\frac{1}{L}$
Ergs-seconds	L^4		
Mass	L^3	Frequency	$\frac{1}{L}$
Energy	L^3		
Work	L^3	Coefficient of resistance	$\frac{1}{L}$
Temperature	L^3		
Magnetic moment	L^3	Reluctance	$\frac{1}{L}$
Volume	L^3		
Electric quantity	L^2	Without dimensions $\left(=\frac{L}{L}\text{ or }\frac{L^2}{L^2}\text{ or }\frac{L^3}{L^3}\right)$:	
Magnetic pole	L^2		
Magnetic flux	L^2	Magnetic permeability	
Magnetic induction	L^2	Dielectric constant	
Area	L^2	Velocity	
Force	L^2	Elasticity	
Moment	L^2	Intensity of electrostatic field	
Power	L^2	Intensity of magnetic field	
Gravitational permeability	L^2		
Angular acceleration	$\frac{1}{L^2}$	Constants of nature:	
		a (black radiation density)	$\frac{1}{L^3}$
Time	L	c_2 (cm. degrees)	L^4
Period	L	h (ergs-seconds)	L^4
Length	L	c (cms. per second)	1
Current	L	$\frac{e}{m_e}$ (charge to mass)	$\frac{1}{L}$
Coefficient of self induction	L	k_2 (ergs/degrees)	$\frac{1}{L}$
Electromotive force	L	n (Rydberg frequency)	$\frac{1}{L}$
Capacity	L	m_e (mass of electron)	L^3
Magnetomotive force	L	m_H (mass of H atom)	L^3
Acceleration	$\frac{1}{L}$	e (electron charge)	L^2

It will be observed that magnetic quantity $\frac{L^3}{T}$ now becomes L^2, and has the dimensions of electrical quantity, (e). Both are L^2. And this is the same as the dimensions of force and also of gravitational permeability. Now the only way we recognize magnetism or electricity is by the force it exerts on charged bodies. It is, therefore, extremely interesting that all three are nothing but force, L^2. Magnetic induction and magnetic pole are also L^2.

Furthermore electric charge being (L^2), shows that it cannot be simple, but must be constituted by the product of two coordinates (L_1) and (L_2). It is an interesting question what these two coordinates are and whether electric quantity, or e, is in reality (L^2), or (L^2), or (L_1) (L_2). It should be possible in the latter case to make an electron charge either by an increase in L_1 with less L_2, or vice versa. Just as mass can be formed by much L^2 and little L_1, or by a smaller amount of electricity and a greater inductance. That is, we may have the same quantity of mass by moving a large number of electrons slowly; or a smaller number with greater velocity. Similarly, e may be formed by moving a long distance along L_1 and a small distance along L_2, or a small interval along L_1 and a large interval along L_2, or by equal intervals of both. The square root of e in e. s. u. is 2.185×10^{-5} units of interval or L.

Mass, energy, and moment of magnetism now all have the same dimensions. They may be identified as equivalent, provided the various spatial coordinates are equivalent, but not if they are not. Each has the dimensions of L^3. Again it is obvious that each of these three has in it electric quantity, L^2, and self inductance, L. Furthermore if these various coordinates are not the same, we can obtain the same quantity of mass or energy or magnetic moment by changes in velocity or in any of the three intervals.

Velocity, elasticity, specific inductive capacity, magnetic permeability, intensity of magnetic field, intensity of electrostatic field, are now without dimensions. They are but the ratio of two intervals L_x/L_y.

Acceleration, angular velocity, frequency, coefficient of resistance and reluctance all have the dimensions of the reciprocal of an interval, $\frac{1}{L}$.

Time, period, length, current, coefficient of self induction, electromotive force, capacity, magnetomotive force, all have the dimension L.

Moment of inertia is L^5.

The interpretation of these results may be left for each one to make for himself. That they offer an enormous apparent simplification of things is obvious. In relativity the attempt has been made already to identify mass with energy. Eddington[6] for example says "Hence it appears that mass (inertia) and energy are essentially the same thing or at the most two aspects of the same thing."

The foregoing resolution of the dimensions of each to L^3, or space,

[6] Eddington: Space, Time and Gravitation. Cambridge, 1920.

clearly indicates this, provided that space is isotropic or that if, when any one coordinate is selected and named one thing, such as time, or self induction, the others are at once determined by their relation to this. Mass and energy have the dimensions of volume. In the terms of the ether this means that the energy of the ether, like its mass, is everywhere the same.

But if time is kept as a separate entity not to be equated with a length, then energy is more than mass. For energy is magnetic flux $\times$ electric quantity per second, and mass is magnetic flux $\times$ time. And mass is certainly more than inertia since mass is the product of three variables, L_1, L_2, L_3, whereas inductance has but one, L.

The bearing of these results on the numerical relations of the constants of nature will be considered in a following paper.

ERRATUM

Vol. 13, No. 10, May 19, 1923, page 211, third paragraph from top of page, lines 4 and 5: The expression "Type species *H. coronatus* n. sp." should be deleted.

ZOOLOGY.—*An amendation of* Hoplolaimus *Daday, 1905, nec auctores.* N. A. COBB, U. S. Department of Agriculture.

In 1905, Daday proposed the new genus *Hoplolaimus* on the basis of a single female nema from soil in Paraguay; the characterization was necessarily imperfect. Daday could give only the location of the vulva and state his belief that the unseen female organs were double. The mouth parts were such that so experienced an observer as Daday readily concluded he had found a new tylenchoid genus. Hence, *Hoplolaimus.*

Hoplolaimus was so imperfectly characterized that numerous subsequent authors have referred to it a variety of species that seem certain not to belong to it, in the light of recent discoveries now to be described.

Hoplolaimus Daday 1905 amend.

Coarsely annuled typical tylenchidae with a prominently set off, lobed lip-region composed of several annules, and an onchium with more or less lobed basal bulbs. -f- and -m. Males with lobed bursa encompassing the tail. Type species *H. coronatus* n. sp.

$$\frac{3.4}{2.4} - \frac{7.8}{2.7} = \frac{12}{3} - \frac{''}{''} - \frac{56.}{3.3} = \frac{''}{''} - \frac{98.4}{2.4} \quad 1.6''$$

H. coronatus n. sp.

The transparent colorless layers of the naked cuticle are traversed by plain, transverse striae, all alike, about three microns apart and easy of resolution, which are not further resolvable, and which are altered materially on the lateral fields by the presence of *three longitudinal wings*, occupying a space, measured midway on the nema, one-third as great as the width of the body. The optical expression of these wings consists of four parallel lines, of which the two outer are rather distinctly crenate, while the two inner, occupying a little less than a third of the wing space, connect with refractive cuticular alterations of the striae, which thus give rise to a rather distinct more or less *quadrangular network* on the lateral fields.

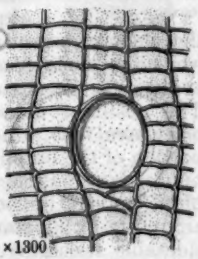

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FIG. 1. In the left lateral chord (usually) of the male Hoplolaimus at lat. 86, and in the right lateral chord of the female at lat. 32, there is an interesting unpaired spheroidal amphid-like organ, one-fourth as wide as the body, having connections fore and aft, and opening outward on the wing region in the form of a depressed near-circle, 8 microns across. See above.

On the neck the wings become reduced to two. The cuticle, about two and one-half microns thick, is striated internally as well as externally. As usual, the annules close to the lip region are somewhat narrower than those farther back. From somewhat behind the anus the final striae on the tail of the female make a gradually smaller angle with the lateral line and finally encircle the terminus in the *lateral plane*. Very slightly oblique longitudinal striae, due to the attachment of the musculature, are visible in most regions of the body. There are no dermal appendages and no series of pores has been seen in the cuticle, but there is an unpaired lateral organ on both the female and male. (See Fig. 1.) The cylindroid neck becomes convex-conoid at the rounded head, which is continuous and presents a central mouth opening only very slightly depressed. The lip region however is a flat, bluntish cone, about twenty microns broad by eight microns high; it is set apart by a *very distinct*

* This type citation to be eliminated. correction slip inserted in # 13 of Seventh ed. of this journal.

FIG. 2. An obligue dorso-ventral view of head of *H. coronatus* n. sp. The quadrangular nature of the cap is shown in the front view, above.

constriction so that it constitutes a sort of *cap on the front of the head.* This somewhat *quadrate cap is longitudinally faintly six-lobed* and each lobe is again longitudinally as well as transversely subdivided. See Fig. 2. The lip region of the male is like that of the female except that it is more nearly hemispheroid, that is, relatively higher and slightly larger. There is a rather robust six-ribbed, yellowish, dome-like structure as the framework of the lip-region, through it is obscured by the nature of the cuticular covering. This framework extends a little back of the labial constriction and its yellow color becomes more evident here. See Fig. 3. Needless to say, therefore, the amalgamated lips are fixed, and shut closely around the anterior extremity of the onchium. Whether the lip region is innervated remains unknown, but no innervations have been seen. The tylenchoid *pharynx is of a very robust nature* and reminiscent of that of *Nemonchus.* The base of the onchium, or spear, thirteen microns wide by ten high, is very distinctly three bulbed and is about one-fifth as wide as the corresponding portion of the head. *Each of the three bulbs is anteriorly somewhat "lobed,"* presenting sometimes two and sometimes three rather distinct forward pointing knobs. Owing to its index of refraction this lobed base of the onchium, as well as the "hilt," are almost totally invisible in balsam mounts; while the acute tapering anterior part remains distinctly visible,—another evidence of the two-fold character (and origin) of the tylenchoid onchium. The posterior attachment of the musculature comprising the *ellipsoidal pharyngeal bulb,* which is one-half as wide as the head, is not only to the front portion of the bulbs of the onchium but also to their posterior surfaces. The hilt is about half as wide as the bulbous base; the anterior end of the spear is blunt, and the lumen relatively unusually narrow;—all which makes the spear an unusually substantial structure, capable of puncturing tissues offering considerable resistance. In harmony with this, the ellipsoidal to obpyriform *spear guide is of strong construction,* consisting in part of six outwardly bowed elastic elements surrounding the anterior third of the spear and springing backward from the base of the cutinized lip region. (Figs. 2 and 3.) This six-fold spear guide has a variable length and width, its form changing with the attitude of the spear;—when at rest, with a length of fourteen microns, its width is about eight microns,—that is, it is about one-third as wide as the corresponding portion of the head. In addition to this spear guide the cutinized lip-region fits closely around the anterior portion of the spear for a considerable distance. Both the spear and the spear guide appear to present traces of transverse striation corresponding in fineness with the minute subdivisions of the annules sometimes visible in the subcuticle. The two parts of the spear are rather distinctly set off from each other by a very fine transverse junction mark, as in many *Tylenchi.* No amphids, deirids or phasmids have been seen. There are no eyespots. The oesophagus is tylenchoid, presenting however, as already indicated, a rather distinct pharyngeal bulb, something rather uncommon in the *Tylenchidae.* The spherical or oblate median oesophageal bulb is half as wide as the corresponding portion of the neck, and is set off both fore and aft from the remaining narrow portions of the oesophagus,—very abruptly behind and rather abruptly in front. Behind the median bulb the narrow oesophagus gradually enlarges to form a rather obscure posterior

long-clavate swelling which at its widest part is one-fourth as wide as the base of the neck. It is however natural to imagine the swollen salivary glands to be joined with this inconspicuous posterior portion of the oesophague and thus at first to get an idea that the posterior part of the oesophagus comprises a wide clavate swelling, three-fourths as wide as the base of the neck. We may say therefore that the oesophagus behind the pharynx is about one-seventh, at the nerve-ring also about one-seventh, and finally about one-fourth as wide as the corresponding portion of the neck. The lining of the oesophagus is a distinct feature arteriorly and consists of a narrow highly refractive tube; posteriorly the lining is inconspicuous. The musculature is fine and colorless. There are *three salivary glands* clustered opposite the posterior two-fifths of the oesophagus. One of these cells comprises the anterior portion of the cluster, while the other two may lie more or less opposite each other as the posterior part. The salivary glands are very well developed, and two of them, submedian, empty through ducts into the base of the valve of the median bulb, while the third, the dorsal, passes forward and empties into the dorsal side of the oesophageal lumen not far behind the base of the onchium. At the mouth of each duct *a faint ampulla* is usually visible. The median oesophageal bulb presents a spheroidal, simple, strongly refractive valve one-sixth as wide as the bulb itself. There is no distinct cardia. The thick-walled intestine, which presents a distinct refractive lumen, is not set off from the oesophagus by a cardiac collum, the change from oesophagus to intestine being gradual. The intestine has its cells closely packed with granules of variable size, the largest being one-twelfth to one-sixteenth as wide as the body. These colorless, non-birefringent, spherical granules *gradually decrease in number* in the cardiac region and cease altogether opposite the posterior portion of the salivary glands; they are sometimes so arranged as to give rise to a tessellated effect. The intestine somewhat gradually becomes four-fifths as wide as the body, and is composed of cells of such a size that two to three are presented in each cross section. The cells of the intestine are so closely packed with granules as to make it difficult to examine successfully in living specimens details of the anatomy of adjacent organs. The exceedingly small *anus and the rectum are very inconspicuous.* There is no pre-rectum. The *tail is of an elongate hemispheroid form,* very broad and rounded at the extremity, and symmetrical. There is no spinneret and there are no caudal glands. Measured at the latitude of the nerve-ring, the lateral chords are as wide as the cuticle is thick, or wider; somewhat farther back, they are about one-third as wide as the body. They contain scattered colorless refractive spherical granules of variable size, considerably smaller than those of the intestine. Behind the base of the neck, at a distance about equal to the diameter of the body, there is a cell which presses the intestine well to one side. This cell is about as long as wide but not spheroidal. It is finely granular, one-half as wide as the body, and seems very probably to be connected with the renette. The *excretory pore,* which is opposite the base of the neck, is rather distinctly to be seen, though of small size. It lies between two annules, and the nearer striation swerves a little to one side for it. The renette duct leads *inward and to the right* along the right lateral chord. From the somewhat depressed, rather large and rather conspicuous vulva, which is a transverse slit two-fifths as long as

FIG. 3. Nearly lateral view of *H. coronatus* n. sp. Below, a semi-contour of the body in the same region, showing the double wing.

the body is wide, the strongly cutinized medium sized vagina extends inward at right angles to the ventral surface half the way across the body. From toward the ends of the valvular opening *four muscles pass obliquely* to the ventrally submedian regions of the body, two forward and two backward,—two to the right and two to the left. Furthermore there are two *transverse* valvular muscles attached near the ends of the opening and fanning out to the lateral parts of the body wall, one right and the other left, each partially encircling the intestine. Each of the two outstretched uteri, about two body-widths long and about one-fourth as wide as the body, at its distal extremity presents a spermatheca three-fourths as wide as the body, sometimes containing what appear to be toward one hundred sausage shaped sperm cells each sometimes having a bunch of chromosomes at one end. These spermatheca are located at a distance from the vulva two or three times as great as the diameter of the body. In the late autumn they are a very uniform feature of the adult females which have depcsited no, or very few, eggs. As thus far seen, the slender outstretched ovaries are about one-fourth as wide as the body; both lie on the left side. They are narrow and somewhat cylindroid and contain one hundred or more ova arranged somewhat irregularly.

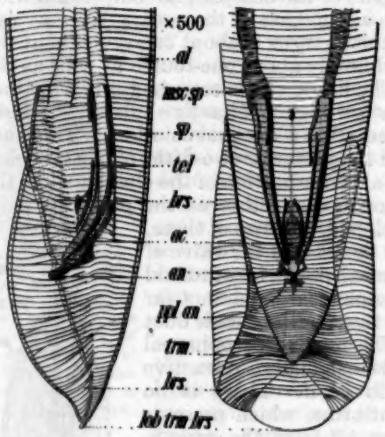

FIG. 4. Lateral and ventral views of the tail end of the male of *Hoplolaimus coronatus* n. sp. Treated with potassium hydrate to obliterate non-cutinized structures.

$$\frac{3.4}{2.4} - \frac{8.9}{2.8} = \frac{13.}{3.2} - \frac{47}{N} - \frac{97.5}{3.5} \quad 1.4\text{--}$$

The male is like the female in form. The spicula are colorless. A portion of the gubernaculum lies ventrad,—(telamon of Hall). See Fig. 4. There are no preanal ventral supplementary male organs, and no ribs occur in the bursa. The striae of the bursa on the ventral side are less distinct near the ventral line. The terminal lobe of the bursa appears destitute of striation; if any striae are present they must be exceedingly fine. The vas deferens appears to be about one-half as wide as the body. The narrow cylindroid testis tapers a little, and at the blind end is only one-fourth as wide as the body. The granular sperm cells seen in the vas deferens are about one-tenth as wide as the body; the spermatocytes, farther forward, one-eighth.

Habitat: Found in soil immediately about a Mermis "nest," (*Agamermis decaudata*), Four Mile Run, Falls Church, Va., U.S.A. Nov., 1922.

The movements of this nema are very slow. The limber body readily takes on sharp sigmoid curves and is sometimes seen coiled; in fact the males can coil rather closely. From this amended characterization it seems evident the *Hoplolaimus* Daday (not of other authors) is a rather clearly marked genus. The closest relative of *Hoplolaimus is Dolichodorus* Cobb 1914.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

PHILOSOPHICAL SOCIETY

878TH MEETING

The 878th meeting was held in the Cosmos Club Auditorium on Saturday, February 10, 1923, with President White in the chair and 43 persons in attendance.

Mr. G. W. LITTLEHALES addressed the Society on *New resources to lighten the labor of navigators in finding geographical position from observations of celestial bodies.* The paper was illustrated by lantern slides and was discussed by Messrs. RUDE, LIFEROCK, and PETERS.

Author's abstract: Included in the equipment of navigators are instruments for measuring the altitude of celestial bodies above the horizon and also chronometers or watches intended to show, from instant to instant, the time of day at the meridian of Greenwich.

An observer who has determined the altitude of a celestial body at a given instant of time has in reality located himself at the end of a radius whose length is the zenith distance, or 90° minus the altitude of the celestial body, and whose origin or center is the geographical position of the observed celestial body, or that place on the surface of the earth which is vertically under the observed body at the instant of observation. In the absence of knowledge of the precise direction of the radius, the only definite information to be obtained from the observation of the altitude of a celestial body at a given instant is that the observer is located on the circumference of a circle described by the radius.

Each separate altitude and corresponding Greenwich time of observation, whether it be of a different celestial body or of the same celestial body in a different quarter of the heavens, will result in one of these circles of position; and it is by the intersection of the circumferences of two or more of these circles that the actual geographical position of the observer is fixed.

Of course, if the observer has migrated in the interval between two observations, it will be necessary, in order to find his geographical position at the instant of the second sight, to consider the center of the first circle of position to be moved in a direction represented by his true course between the stations and by an amount equal to the distance between them.

These circles are called Sumner circles or circles of equal altitude since the circumference of each traces out a line on the earth's surface from every point in which the altitude of the observed celestial body is the same at the time of observation. In practice it is unnecessary to draw the whole of the circumference corresponding to each observation.

Every part of the Sumner circle is perpendicular to the true bearing of the celestial body observed, and therefore the azimuth of the body observed is equal to the angle which the Sumner line makes with the parallels of latitude on the Mercator chart. Hence, if the latitude and longitude of one point in the Sumner line be known, and also the true azimuth of the observed position, the line may be drawn on the chart.

The process by which the longitude corresponding to a given latitude, or rather the relative longitude or hour angle, and also the azimuth are calculated is simply the solution of the spherical triangle whose sides are the estimated co-latitude, the zenith distance or co-altitude, and the polar distance or co-declination. These three sides are the data: the results to be calculated are the hour angle of the celestial body from the ship's meridian, and the azimuth.

It is evident that if the solutions of these two angles are tabulated corresponding to all the usual values of each of the three sides of the triangle, there will be no need for any calculation, and, hence, that a simplification is afforded in obtaining the data for drawing the Sumner line by the new *tables of simultaneous hour angles and azimuths* in which are given the values of the hour angle and azimuth that a celestial body would have at stated true altitudes above the horizon of an observer situated in successive degrees of latitude ranging from 60° north to 60° south of the equator.

Mr. C. V. HODGSON presented a paper on *Precise measurement of distances on the earth.* The paper was illustrated by lantern slides and was discussed by Messrs. WM. BOWIE, RUDE, PAWLING, TUCKERMAN, HUMPHREYS, HAWKESWORTH, FERNER, and GISH.

Author's abstract: The first part of the paper was a brief résumé of the earlier methods of precise distance measurements with reference to the approximate accuracy obtained with each. Lantern slides were shown of the contact bar, duplex bar, wire bar and tapes, with references to the principal mechanical and theoretical features of each, and their advantages and disadvantages.

A recent extension of the principles of precise base measurement to precise traverse was then discussed. About 3000 miles of precise traverse has been executed in the United States by the Coast and Geodetic Survey in regions where triangulation would be very expensive. The accuracy of this traverse, as measured by closure in position, is usually from 1–70,000 to 1–100,000. The cost will range from $38.00 to $60.00 per mile, and the speed with the single party from 60 to 120 miles per month.

As an indication of the accuracy which may be obtained with invar tapes, reference was made to a precise base being measured in California where the accuracy desired is represented by an actual error of 1 part in 1,000,000 with a probable error of about 1 part in 4,000,000. Such accuracy necessitates unusual refinement in methods, some of which consist of special standardizations of tapes and bars, corrections for the temperature of the springs and the balances, investigations as to the amount of error resulting from slight errors in tension, temperature and inclination of tapes, and special field methods to limit such errors.

Mr. R. W. G. WYCKOFF presented two papers, (1) on *Atomic radii*, and (2) on *Crystal structure of the alums.* The papers were illustrated by lantern slides and were discussed by Messrs. TUCKERMAN, WILLIAMSON, SOSMAN, and GISH.

Author's abstract: A comparative study has been made of the best available crystal structure data to see whether they are in agreement with the hypothesis of constant atomic radii advanced two years ago by W. L. Bragg. According to this hypothesis if a definite size is assigned to the atoms of the various elements, crystals can be built up by packing together these atomic spheres. The existing data show clearly that this idea is not satisfied by the better information now at hand. They do, however, conform quite exactly to the rule that in isomorphous crystals composed of only two kinds of atoms the interatomic distances have additive properties which can be illustrated through a summing up of "atomic radii." The rule often holds approximately between the most electropositive and electronegative atoms of a crystal but deviates widely in its application to the other atomic separations. In crystals of different sorts the effective volume of an atom depends both upon the nature of the other atoms with which it is associated and the manner of their distribution about it. These data seem to show furthermore

that the more nearly alike is this atomic environment in the two cases, the closer together are the "atomic radii" of an atom occurring in both.

Author's abstract: Laue photographic studies upon potassium and ammonium aluminum alums show that the previous X-ray spectrometer data (Vegard and Schjelderup) are incorrect and assign an incorrect atomic arrangement. A comparison of the spectrometer measurements with some spectrographic observations proves that the spectrometer inaccuracies arise from the entrance of reflections from other than the principal reflecting face into the ionization chamber. This difficulty is pointed out to be one inherent in the spectrometer technique itself so that the alums thus furnish an excellent and clear-cut illustration of the insufficiency of the original spectrometer procedure for the determination of the structures of crystals using X-rays.

The unit cube of the correct structure contains four molecules of the composition $RAl(SO_4)_2 \cdot 12HO_2$, where R is either K or NH_4. The 12 water molecules fall into two groups of six each. The hydrogen atoms in the ammonium group present an interesting difficulty in the impossibility of arranging them into a chemically plausible radical which will possess a symmetry in keeping with that of the crystal as a whole. For these crystals the corresponding space group is T_h^6.

879TH MEETING

The 879th meeting was held in the Cosmos Club Auditorium on Saturday, February 24, 1923. It was called to order by President White with 24 persons present.

Owing to some delay at the projecting lantern, opportunity was afforded for informal communications. Mr. I. G. PRIEST presented an informal communication on a new variation in the use of the Nicol prism. Dr. W. J. HUMPHREYS presented an informal communication on a correct explanation of the diffraction phenomenon commonly known as the "glory" or "Brockenbow."

Dr. C. G. PETERS presented a paper on *Changes in the index of refraction of glass at high temperature.* The paper was illustrated with lantern slides and was discussed by Messrs. WILLIAMSON, GISH, HUMPHREYS, WASHBURN, HAWKESWORTH, WHITE, and SOSMAN.

Author's abstract: Using an interference method the changes in the refraction of nine different kinds of optical glass were measured over the temperature interval 20° to 650°C.

The samples made in the form of a plate having two faces parallel were placed between two fused quartz mirrors and heated in an electrical furnace. The index at any temperature can be represented by the relation

$$\mu_t = \frac{N_g + \Delta N_g}{N_\nu + \Delta N_\nu}$$

Where N_ν is the number of light waves between the fused quarz plates at 20°C., N_g the number of waves in glass between the two parallel surfaces of the plate at the same temperature, ΔN_ν the number of fringes that pass the reference mark on the quartz plates, and ΔN_g the number that pass reference mark on the glass plate while the temperature is increased to t.

The index of each glass increased with the temperature until the annealing range was reached which was near 500°C. for these glasses. As the temperature was still further increased, the index decreased rapidly. The rapid

decrease in the index between 500 and 650°C. is due to the decrease in density caused by the rapid increase in the expansivity of glass in this region.

It is usually assumed that the index changes according to the relation $\frac{\mu - 1}{d} = C$ or a similar relation. With glass, however, the index increased between 20 and 500°C., while the density decreased. This increase in index must be due to the same influence that causes the ultra-violet absorption band to move toward the longer wave-length region when glass is heated. The effect of this cause must be the difference between the measured value for the index and the value computed from the density relation. This was found to increase at a constant rate even through the critical region between 500 and 600°C.

By invitation, Mr. C. C. KIESS presented a paper on *Regularities in the sprectra of chromium and molybdenum.* The paper was illustrated with lantern slides and was discussed by Messrs. MOHLER, HAWKESWORTH, MEGGERS, SOSMAN, and HUMPHREYS.

Author's abstract: The elements in whose spectra series have been found fall, with a few exceptions, in the first three columns of the Periodic Table. The unravelling of series relationships in the spectra of the other elements has been delayed partly because of the complexity of their spectra but chiefly by the lack of reliable wave-length data in the spectral regions to the red of wave-length 6000 A. Recent studies of the arc spectra of chromium and molybdenum at the Bureau of Standards have led to the discovery of series and other regularities in them. The same types of series exist in both spectra. In each spectrum, there are two systems of series whose members are widely separated triplets. Each system consists of a principal, sharp, and diffuse series, the members of which are separated from the homologous members of the second or parallel system by constant frequency differences. For chromium the characteristic triplet separations are $\Delta\mu_1 = 112.4$, $\Delta\mu_2 = 81.4$ and $\Delta\nu_1' = 115.1$, $\Delta\nu_2' = 91.4$; while for molybdenum the corresponding data are $\Delta\nu_1 = 448.5$, $\Delta\nu_2 = 257.5$ and $\Delta\nu_1' = 379.9$, $\Delta\nu_2' = 233.4$. In addition to the wide triplets, each spectrum contains narrow triplets of which the separations are $\Delta\sqrt{}_1 = 8.8$, $\Delta\sqrt{}_2 = 5.6$ for chromium, and $\Delta\nu_1 = 121.5$, $\Delta\nu_2 = 87.0$ for molybdenum. Besides the series regularities there exist in each spectrum other regularities known as multiplets. These consist of groups of nine or twelve lines linked together by constant frequency differences. Since more lines of the spectra of these elements have been assigned to multiplets than to series, it would seem to indicate that the multiplet rather than the series type of regularity is the predominant one in complex spectra.

880TH MEETING

The 880th meeting was a joint meeting with the Washington Academy of Sciences held in the Cosmos Club Auditorium on Saturday, March 10, 1923.

President VAUGHAN of the Washington Academy of Sciences took the chair with 90 persons in attendance.

Dr. BRIGGS introduced Professor A. SOMMERFELD of Munich, who addressed the joint meeting on the subject *Evidence for the theory of relativity afforded by atomic physics.* The address was illustrated with lantern slides and was discussed by Messrs. BAUER, MOHLER, HAWKESWORTH, and FOOTE.

On motion of Dr. BRIGGS, the joint meeting accorded Professor Sommerfeld a rising vote of thanks for his address.

881ST MEETING

The 881st meeting was held in the Cosmos Club Auditorium on Saturday, March 24, 1923. It was called to order by Vice-President Hazard with 50 persons in attendance.

Mr. J. PAWLING presented a paper on *The 9" Transit of the U. S. Naval Observatory and its work.* The paper was illustrated with lantern slides and was discussed by Dr. HUMPHREYS.

Mr. H. A. MARMER presented a paper on *The tidal phenomena in New York harbor.* The paper was illustrated with lantern slides and was discussed by Messrs. HUMPHREYS and GISH.

Author's abstract: In New York Harbor the tidal and current phenomena exhibit more than usual variety because of two geographic features that distinguish the harbor: (a) Unlike other harbors which generally consist of a tidal bay or river New York Harbor comprises a system of five tidal highways that connect with the open sea by means of two inlets many miles apart; (b) the waterways forming the harbor—Upper Bay, Kill van Kull, East River, Harlem River and lower Hudson River—are either intercommuricating or lead into other bodies of water.

In Upper Bay and in the lower Hudson the tidal and current phenomena are typical of those found in bays and rivers in which the tidal movement is of the progressive-wave type. The time of tide becomes later in going up stream at a rate dependent on the depth, the formula being approximately $r = \sqrt{gd}$, where r is the rate of advance, g the acceleration of gravity and d the depth of the waterway. The mean range of the tide decreases in going upstream, and the strength of current comes about the times of high and low water.

In the East River, the tide changes by $3\frac{1}{2}$ feet through the stretch of 14 geographic miles but not at a uniform rate. The range of tide is 4.4 feet at the eastern end and 7.2 feet at the western, but in the river near the western end is a region with a range of 4.0 feet. The strength of the current is very nearly simultaneous throughout the river.

It has been customary to ascribe the tidal and current phenomena in the East River to the interference of two tide waves, the one coming from Long Island Sound and the other from Upper Bay. But these phenomena can easily be derived by considering the matter from the hydraulic point of view and a slope-line diagram brings out the principal tidal and current phenomena immediately.

The tidal and current phenomena in the Harlem River and in Kill van Kull, as in the East River, are conditioned by the fact that the tidal movement in these waterways is largely of the hydraulic type.

A brief description was given of a bifilar suspension direction indicator used in determining the direction of subsurface currents.

Dr. F. E. WRIGHT presented a paper on *Methods for distinguishing between natural and cultivated pearls.* Specimens of both natural and cultivated pearls were exhibited, and the paper was discussed by Messrs. TUCKERMAN, IVES, RUDE, and HUMPHREYS.

Author's abstract: The cultivated pearl from Japan consists of a center or nucleus of mother-of-pearl or of an inferior pearl, on which thin concentric shells of pearly substance have been deposited by the shell secreting epidermis of the pearl oyster; it differs from the normal fine pearl of commerce only in the fact of its foreign nucleus; the fine pearl consists of the thin concentric shells of the pearly substance throughout. Any method for

distinguishing between cultivated and fine pearls must be based on some property that brings out the inhomogeneity between the foreign nucleus and the pearl periphery in the cultivated pearl. If the nucleus is a bead of mother-of-pearl the fact that the mother-of-pearl reflects the light strongly along a direction normal to the pearly layers and weakly along other directions can be used as a criterion. Under proper conditions of illumination the characteristic mother-of-pearl sheen can be seen shining out as a subdued glow from the nacre bead of a cultivated pearl when held in certain positions. In transmitted light the corresponding differences in degree of transparency for directions normal and parallel with the pearly layers of the nacre bead can be used to reveal its presence. Under intense illumination the banding of the mother-of-pearl nucleus gives rise to a series of bright lines on a dark field when the pearl is held in a certain position. Intense illumination is obtained by use of a strong artificial light or the sun focussed by means of a condenser lens on the pearl. The pearl may be examined in air or immersed in a refractive liquid. Polarized light was found to be of no assistance except for cutting out glare due to extraneous light.

A third method is a modification of the method of Galibourg and Ryziger for examining the walls of the hole drilled through the pearl. In place of the mercury column which they employed a bead made by fusing the end of a pure gold wire 0.2 mm. diameter is used as a reflecting mirror. By its use any irregularities or changes in the substance lining the hole in the pearl can be seen reflected by the gold bead when examined under proper conditions of illumination through a low power microscope or binocular.

882ND MEETING

The 882nd meeting was held jointly with the Washington Academy of Sciences, the Washington Society of Engineers, and the American Society for Steel Treating in the auditorium of the Interior Building on Saturday, March 31, 1923.

The chair was taken by President WHITE of the Philosophical Society with 150 persons in attendance.

Dr. WALTER ROSENHAIN, F. R. S., of the National Physical Laboratory, addressed the meeting on *The structure and constitution of alloys.* The address was illustrated with lantern slides and was followed by discussion.

J. P. AULT, *Recording Secretary.*

SCIENTIFIC NOTES AND NEWS

The National Academy of Sciences met at the National Museum April 23–25. On the evening of April 23 an address was given by Dr. W. W. CAMPBELL: *Résumé of results obtained by the Crocker Eclipse Expeditions from Lick Observatory.* A reception followed in the galleries of the Museum.

The annual meeting of the American Geophysical Union was held at the Carnegie Institution on April 17–19. The following sections held meetings: Geodesy, Seismology, Meteorology, Terrestrial Magnetism and Electricity, Oceanography, Volcanology, and Geophysical Chemistry.

At the Bureau of Standards Physics Club on Wednesday, April 18, Professor R. A. MILLIKAN spoke on *The penetrating radiations of the upper air.*

Dr. L. SILBERSTEIN lectured on *The helium atom* at the Bureau of Standards, Monday, April 23.

The following two meetings were held in the Assembly Room of the Carnegie Institution. Thursday, April 19. Program: R. A. MILLIKAN: *Present problems in the field of atomic structure and their bearing upon the nature of ethereal radiations.* Wednesday, April 25. Program: A. A. MICHELSON: *Application of interference methods to astronomy.*

In connection with the annual meeting of the Association of Scientific Apparatus Makers, held in Washington on April 20, there was an exhibit of apparatus in the Industrial Building, Bureau of Standards. A number of manufacturers displayed some of their newer developments.

Dr. SVEN HEDIN, the Swedish explorer, gave a talk in the National Museum on Wednesday, April 18, on *Discoveries in Eastern Turkistan and Southern Tibet.*

Dr. GEORGE KIMBALL BURGESS has been appointed director of the Bureau of Standards, to succeed Dr. S. W. STRATTON. Dr. Burgess entered the Bureau as assistant physicist in 1903, was associate physicist from 1905 until 1913, when he became chief of the Division of Metallurgy, which position he has held up to the present time.

At the meeting of the American Philosophical Society in Philadelphia on April 11 the John Scott medal and prize were awarded by the City of Philadelphia to Dr. ARTHUR L. DAY, director of the Geophysical Laboratory, Carnegie Institution of Washington, for his work in the interpretation of geological phenomena and for producing optical glass.

Mr. NEIL M. JUDD, curator of American Archeology, U. S. National Museum, will leave Washington May 1 to resume direction of the National Geographic Society's expedition for the exploration of Pueblo Bonito, one of the largest and best preserved prehistoric ruins in the southwestern United States.